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Symmetry breaking and the Goldstone theorem in de Sitter space

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We consider an $O(N)$ symmetric scalar field model in the mean field approximation and show that the symmetry can be broken (down to $O(N-1)$) in de Sitter space. We find that the phase transition is of a first order, and that its strength depends non-analytically on the parameters of the model. We also show that the would-be Goldstone bosons acquire a mass, effectively becoming pseudo-Goldstone bosons. Our results imply that topological defects can form during inflation.

I. INTRODUCTION

Ever since Kirzhnits and Linde [1] pointed out that thermal radiative effects can induce phase transitions in the early Universe, phase transitions have played a central role in the early Universe cosmology. In particular, they have been used to drive out-of-equilibrium phenomena which can lead to creation of the matter-antimatter asymmetry, preheating, formation of topological defects, *etc.* The effects induced by particle creation in an expanding Universe setting are quite delicate and have not yet been fully understood, albeit there is a large literature on the subject, a non-representative sample includes Refs. [2–17]. Based upon a mean field analysis of a scalar self-interacting theory, Ford and Vilenkin [18, 19] pointed out a long time ago that the infrared effects in de Sitter space may restore symmetries spontaneously broken by the vacuum. However, a crucial sign error invalidated the conclusions of Ref. [18]. The infrared effects in (quasi-)de Sitter spaces have received a considerable attention in recent literature, and several papers have been published [20–24] which have – just as Ford and Vilenkin – treated the problem in the mean field approximation (see also Refs. [25, 26] for some recent mean field results on flat space). In particular, the above-mentioned sign error has been corrected in Refs. [23, 24]. The current consensus is that the infrared

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effects in de Sitter space are strong enough to restore the broken $O(N)$ symmetry. While this is correct if the corresponding effective action is averaged over large (super-Hubble) distances, from the observational point of view the more relevant question is whether the symmetry gets broken or restored when averaged over some fixed physical scale [31]. In this work we take the point of view that the effective action should be averaged over some fixed physical scale and we show that symmetries are then generally not restored in de Sitter space. We also give a simple criterion for symmetry restoration. For simplicity, we consider here only the global $O(N)$ symmetric scalar field model.

For pedagogical reasons we begin in section II by analysing a real scalar field ($O(1)$ model). The central part of the paper is section III where we analyse an $O(N)$ symmetric model on de Sitter space. Section IV is reserved for a discussion, and the Appendix for technical details on the de Sitter space propagator of a scalar field.

II. A REAL SCALAR FIELD

The free action of a real scalar field $\phi(x)$ in D space-time dimensions reads,

$$S[\phi] = \int d^D x \sqrt{-g}\left[-\frac{1}{2}g^{\mu\nu}(\partial_\mu\phi)(\partial_\nu\phi) - \frac{1}{2}m_0^2\phi^2 - \frac{\lambda}{4!}\phi^4\right], \tag{1}$$

where m_0 and λ denote the field mass and quartic coupling, respectively, $g_{\mu\nu}$ is the metric tensor, $g^{\mu\nu}$ its inverse, and $g = \det[g_{\mu\nu}]$. The metric signature we use is $(-,+,+,..)$. The vacuum is given by $\phi^2 = 0$ for $m_0^2 > 0$ and $\phi^2 = -6m_0^2/\lambda$ when $m_0^2 < 0$. In the case when $\phi^2 > 0$ the Z_2 symmetry ($\phi \to -\phi$) of the action (1) is (completely) broken by the vacuum in which $\phi^2 = -6m_0^2/\lambda$. If one goes through a quench from a high temperatures ($e.g$ in an early Universe setting), when $m_0^2 < 0$ domain walls form by the Kibble mechanism [27], such that the state spontaneously breaks translation invariance. By causality, at least of the order of one domain wall forms per Hubble volume. Once formed, their energy density scales as $\propto 1/a^2$ (a denotes the scale factor), such that in decelerating spacetimes domain walls will dominate the energy density at late times (a menace to get rid of), while in accelerating spacetimes (such as inflation) they get diluted.

Here we shall perform a mean field analysis, for which the effective potential (up to two loop order) is of the form,

$$V_{\rm MF} = V_0 + \frac{1}{2}m_0^2(\phi^2 + \imath\Delta(x;x)) + \frac{\lambda}{4!}\left(\phi^4 + 6\phi^2\imath\Delta(x;x) + 3[\imath\Delta(x;x)]^2\right) + \frac{\imath}{2}{\rm Tr}\ln[\imath\Delta(x;x)], \tag{2}$$

where m_0^2 denotes a bare mass term, $\lambda > 0$ a quartic coupling, $\phi(x) = \langle\Omega|\hat{\phi}(x)|\Omega\rangle$ is a mean field, $|\Omega\rangle$ is a state, and $\imath\Delta(x;x') = \langle\Omega|T[\delta\hat{\phi}(x')\delta\hat{\phi}(x)]|\Omega\rangle$ is the Feynman propagator for the field fluctuations

$\delta\hat{\phi}(x) = \hat{\phi}(x) - \phi(x)$, where T stands for time ordering. For simplicity, we have assumed that gravity is nondynamical and that all quantities $\phi(x)$ and $\imath\Delta(x;x)$ are either constant or adiabatically varying in time (on a de Sitter background). Varying the mean field action

$$S_{\rm MF}[\phi,\Delta] = \int d^Dx\sqrt{-g(x)}\left[-\frac{1}{2}(\partial_\mu\phi)(\partial_\nu\phi)g^{\mu\nu} + \frac{1}{2}[\Box_x\imath\Delta(x;x)]_{x'\to x} - V_{\rm MF}(\phi,\Delta)\right] \tag{3}$$

with respect to $\phi(x)$ and $\imath\Delta(y;x)$ results in

$$\left[\Box_x - m_0^2 - \frac{\lambda}{2}\imath\Delta(x;x)\right]\phi - \frac{\lambda}{6}\phi^3 \;=\; 0 \tag{4}$$

$$\sqrt{-g}\left[\Box_x - m_0^2 - \frac{\lambda}{2}(\phi^2 + \imath\Delta(x;x))\right]\delta^D(x-y) \;=\; \imath[\imath\Delta(x;y)]^{-1}\,, \tag{5}$$

where $\Box_x = (-g)^{-1/2}\partial_\mu g^{\mu\nu}\sqrt{-g}\partial_\nu$ denotes the d'Alembertian ($\Box_x = g^{\mu\nu}\nabla_\mu\nabla_\nu$) as it acts on a scalar quantity. Since the coincident propagator $\imath\Delta(x;x)$ is in general divergent (see the Appendix), these equations need to be renormalised. From the structure of equations (4–5), a mass renormalisation suffices [32]. When viewed as a function of the ultraviolet cutoff Λ, the coincident propagator exhibits power law divergences $\propto \Lambda^{D-2}, \Lambda^{D-4}$, *etc.* [25, 26], which are automatically subtracted in dimensional regularisation. In the special cases when the spacetime dimension is even ($D = 2n$, $n = 1, 2, \ldots$), there is a logarithmic divergence in Λ, that manifests itself as a simple pole in the coincident propagator, $\imath\Delta(x;x)_{\rm div} \propto 1/(D-2n)$ ($n = 1, 2, \ldots$), see Eqs. (58–62) in the Appendix. When this divergence is absorbed in the bare mass m_0^2, one gets a finite, renormalised mass term,

$$m^2 = m_0^2 + \imath\Delta(x;x)_{\rm div}\,, \quad \imath\Delta(x;x)_{\rm div} = \frac{H^{D-2}\Gamma\left(\frac{D-1}{2}\right)}{4\pi^{(D-1)/2}}\left[\psi\left(\frac{D}{2}\right) - \psi(D-1) - \psi\left(1-\frac{D}{2}\right) - \gamma_E + \frac{1}{D-1}\right], \tag{6}$$

where m^2 can be either positive or negative. Hence, the renormalised, manifestly finite, form of Eqs. (4–5) is,

$$\left[\Box_x - m^2 - \frac{\lambda}{2}\imath\Delta(x;x)_{\rm fin}\right]\phi - \frac{\lambda}{6}\phi^3 \;=\; 0 \tag{7}$$

$$\sqrt{-g}\left[\Box_x - m^2 - \frac{\lambda}{2}(\phi^2 + \imath\Delta(x;x)_{\rm fin})\right]\imath\Delta(x;x') \;=\; \imath\delta^D(x-x')\,, \tag{8}$$

where $\imath\Delta(x;x)_{\rm fin} = \imath\Delta(x;x) - \imath\Delta(x;x)_{\rm div}$ is the finite part of the coincident correlator, *cf.* Eqs. (59–62).

Since ϕ is (by assumption) slowly varying, we have $\Box\phi \approx 0$, and Eq. (8) yields

$$\left[m^2 + \frac{\lambda}{2}\imath\Delta(x;x)_{\rm fin} + \frac{\lambda}{6}\phi^2\right]\phi = 0\,. \tag{9}$$

This is solved by $\phi = 0$ or, when $\phi^2 > 0$, by

$$\phi^2 = -\frac{6m^2}{\lambda} - 3\imath\Delta(x;x)_{\rm fin} > 0\,, \tag{10}$$

At early times $\imath\Delta(x;x)_{\rm fin}$ grows as given in (60–62), reaching at late times the de Sitter invariant limit (59), and the condition (9) becomes,

$$\phi^2 = -\frac{6m^2}{\lambda} - \frac{3\Gamma\left(\frac{D+1}{2}\right)}{2\pi^{(D+1)/2}}\frac{H^D}{m^2_{\rm MF}} > 0\,, \tag{11}$$

where $m^2_{\rm MF} = \partial^2 V_{\rm MF}/\partial\phi^2$ is the mean field mass satisfying the mass gap equation

$$m^2_{\rm MF} = m^2 + \frac{\lambda}{2}\Big(\phi^2 + \imath\Delta(x;x)_{\rm fin}\Big) = \begin{cases} -2m^2 - \lambda\imath\Delta(x;x)_{\rm fin}\,, & \text{if } \phi^2 > 0 \\ \quad m^2 + \frac{\lambda}{2}\imath\Delta(x;x)_{\rm fin}\,, & \text{if } \phi^2 = 0. \end{cases} \tag{12}$$

Note that the mean field mass $m_{\rm MF}$ is also the mass of field fluctuations in the correlator equation (8). When Eq. (59) is inserted into (12) one gets,

$$m^2_{\rm MF} + 2m^2 + \frac{\lambda\Gamma\left(\frac{D+1}{2}\right)}{2\pi^{(D+1)/2}}\frac{H^D}{m^2_{\rm MF}} = 0 \qquad (\phi^2 > 0)\,. \tag{13}$$

This is solved by,

$$m^2_{\rm MF\pm} = -m^2 \pm \sqrt{m^4 - m^4_{\rm cr}}\,, \qquad m^4_{\rm cr} = (\lambda m^2_{\rm MF})\imath\Delta(x;x)_{\rm fin} = \frac{\lambda H^D\Gamma\left(\frac{D+1}{2}\right)}{2\pi^{(D+1)/2}} \qquad (\phi^2 > 0)\,. \tag{14}$$

We see that – when the symmetry is broken, $\phi^2 > 0$ – there is a minimum $|m^2|$ for which the gap equation (14) permits a meaningful (real) solution [33]:

$$|m^2| > m^2_{\rm cr} = \sqrt{\frac{\lambda H^D\Gamma\left(\frac{D+1}{2}\right)}{2\pi^{(D+1)/2}}}\,. \tag{15}$$

In $D = 2, 3$ and 4, $m^2_{\rm cr} = \sqrt{\lambda/(4\pi)}H$, $\sqrt{\lambda/2}H^{3/2}/\pi$, and $\sqrt{3\lambda/2}H^2/(2\pi)$, respectively. There is a simple way to determine the physically correct sign in Eq. (14). In the limit when $H \to 0$ the infrared enhanced fluctuations are absent, such that one should recover the tree level mass, $m^2_{\rm MF} \to -2m^2$. This then implies that the physical mean field mass corresponds to the positive branch in (14),

$$m^2_{\rm MF} = -m^2 + \sqrt{m^4 - m^4_{\rm cr}} \qquad (\phi^2 > 0)\,. \tag{16}$$

On the other hand, when the symmetry is unbroken, $\phi^2 = 0$, Eq. (12) implies,

$$m^4_{\rm MF} - m^2 m^2_{\rm MF} - \frac{1}{2}m^4_{\rm cr} = 0 \tag{17}$$

which is solved by,

$$m^2_{\rm MF} = \frac{m^2}{2} + \sqrt{\frac{m^4}{4} + \frac{m^4_{\rm cr}}{2}} \qquad (\phi^2 = 0)\,. \tag{18}$$

Here we have dropped the solution with a negative sign in front of the square root, because for that solution $m^2_{\rm MF} < 0$, which is unacceptable on physical grounds (in this case the de Sitter invariant

state would be unstable under small perturbations). When the solution (18) is inserted into Eq. (13), one can show that the only real solution for ϕ is $\phi = 0$, consistent with the assumption of unbroken symmetry made in deriving Eq. (18) [34].

To summarise, we have found that, when

$$m^2 < -m_{\rm cr}^2 = -\sqrt{\frac{\lambda H^D \Gamma\left(\frac{D+1}{2}\right)}{2\pi^{(D+1)/2}}} \qquad (\text{vacuum breaks the } \mathrm{Z}_2 \text{ symmetry})\,, \tag{19}$$

the infrared fluctuations on de Sitter space may not be able to restore the broken Z_2 symmetry ($\phi \to -\phi$) of the vacuum of a real scalar field (1). In this case $\phi^2 = 3m_{\rm MF}^2/\lambda > 0$ and Eq. (16) applies. Otherwise, when $m^2 \geq -m_{\rm cr}^2$ the Z_2 symmetry is restored and Eq. (18) applies. From Eq. (16) we see that, in the broken symmetry case, there is a minimum mean field mass, given by $(m_{\rm MF}^2)_{\rm cr} = -m^2 = m_{\rm cr}^2$, implying that for any finite coupling λ, as $|m^2|$ increases from $|m^2| < m_{\rm cr}^2$ to $|m^2| > m_{\rm cr}^2$, the order parameter $\phi^2 = 3m_{\rm MF}^2/\lambda$ will experience a jump,

$$\Delta\phi^2 = \frac{3m_{\rm cr}^2}{\lambda} = \sqrt{\frac{9H^D\Gamma\left(\frac{D+1}{2}\right)}{2\lambda\pi^{(D+1)/2}}}\,, \tag{20}$$

implying that the (phase) transition is of a first order. Notice that increasing H and decreasing λ strengthens the transition.

The above analysis has been performed under the assumption of adiabaticity, *i.e.* the time and spatial derivatives of ϕ have been neglected. Let us have a closer look at the validity of this approximation. When $m^2 < -m_{\rm cr}^2$, the Kibble mechanism necessitates formation of domain walls, which spontaneously break the de Sitter symmetry. The thickness of the domain walls is $L_{\rm dw} \sim 1/m_{\rm MF}$, implying that, when $L_{\rm dw} > 1/H$ ($L_{\rm dw} < 1/H$) the domain walls will be of a super-Hubble (sub-Hubble) thickness. In the former case the adiabaticity assumption for $\phi(x)$ and $\imath\Delta(x;x)$, which was used in the derivation of the mean field results above, may be unjustified because of the large spatial gradients and time derivatives of the fields that can in that case develop. Indeed, from the mean field equation (7), which in general homogeneous expanding spacetimes reads,

$$\left[\partial_t^2 + 3H\partial_t - \frac{\nabla^2}{a^2} + m_{\rm MF}^2\right]\phi - \frac{\lambda}{3}\phi^3 = 0\,, \tag{21}$$

we can estimate the spatial gradients of the fields as $(\nabla/a)\phi \sim m_{\rm MF}\phi$ (an analogous estimate holds for the time derivative, $\partial_t\phi \sim m_{\rm MF}\phi$), implying that one cannot anymore neglect the derivative terms in (21), such that – in this case – a more sophisticated (numerical) analysis is required. Nevertheless, we can make a rough estimate of the critical mass as follows. When $L_{\rm dw} > 1/H$

the superluminal expansion of the space on super-Hubble scales will win over the forces which hold the domain walls together, such that they will stretch together with the expanding de Sitter space, restoring the symmetry. This simple argument suggests that the actual critical mass-squared (relevant in the full dynamical mean field setting (21)), above which the de Sitter symmetry gets restored and below which it gets broken, is given roughly by $-m_{\rm cr}^2 \sim -H^2$, and not by Eq. (19). To get a more precise value however, a more complete (numerical) analysis of Eq. (21) is needed.

III. THE $O(N)$ MODEL

We shall now consider the symmetry breaking in an $O(N)$ symmetric scalar field theory in the early Universe setting. Recall that this model allows for formation of (global) cosmic strings (when $N = 2$), global monopoles (when $N = 3$), global cosmic textures (when $N = 4$), *etc.* The free action of an $O(N)$ symmetric scalar field reads,

$$S = \int d^D x \sqrt{-g} \left[-\frac{1}{2} g^{\mu\nu} \sum_{a=1}^{N} (\partial_\mu \phi_a)(\partial_\nu \phi_a) - \frac{1}{2} m_0^2 \sum_{a=1}^{N} \phi_a^2 - \frac{\lambda}{4N} \left[\sum_{a=1}^{N} \phi_a^2 \right]^2 \right] . \tag{22}$$

Similarly as in the Brout-Englert-Higgs (BEH) mechanism, when $m_0^2 < 0$ and $\lambda > 0$ the vacuum breaks the $O(N)$ symmetry to the $O(N-1)$ symmetry, such that the resulting vacuum manifold is the N dimensional sphere, $S^N \sim O(N)/O(N-1)$ [27]. As a result, one of the scalars acquires a mass, while the other $N-1$ scalars remain massless. These massless excitations are known as Goldstone bosons. Unlike in the simple $O(N)$ model (22), at low energies the Goldstone bosons in the BEH mechanism acquire a mass and become the longitudinal excitations of the $W^\pm$ and Z bosons. For that reason they are known as pseudo-Goldstone bosons. We shall now see that the Goldstone bosons of the $O(N)$ model in de Sitter space (more generally in inflationary spacetimes) become massive due to the infrared (super-Hubble) enhancement of scalar correlations, and in that respect they can be considered as pseudo-Goldstone bosons.

The mean field (two loop) effective potential of an $O(N)$ symmetric field ϕ_a corresponding to the tree level action (22) reads,

$$\begin{aligned} V_{\rm MF} \;=\; & \frac{1}{2} m_0^2 \left[\sum_{a=1}^{N} \left(\phi_a^2 + \imath\Delta_{aa}(x;x) \right) \right] + \frac{\lambda}{4N} \Bigg[\bigg(\sum_{a=1}^{N} \phi_a^2 \bigg)^2 + 2 \bigg(\sum_{a=1}^{N} \phi_a^2 \bigg) \sum_{b=1}^{N} \imath\Delta_{bb}(x;x) \\ & + 4 \sum_{a,b=1}^{N} \phi_a \phi_b \imath\Delta_{ab}(x;x) + \bigg(\sum_{a=1}^{N} \imath\Delta_{aa}(x;x) \bigg)^2 + 2 \sum_{a,b=1}^{N} \Big(\imath\Delta_{ab}(x;x) \Big)^2 \Bigg] + \frac{\imath}{2} {\rm Tr} \ln \Big(\imath\Delta_{aa}(x;x) \Big) , \end{aligned} \tag{23}$$

resulting in the following two loop effective action,

$$S_{\rm MF}[\phi_a,\Delta_{bc}] = \int d^Dx\sqrt{-g}\left[-\frac{1}{2}\sum_{a=1}^{N} g^{\mu\nu}(\partial_\mu\phi_a)(\partial_\nu\phi_a) + \frac{1}{2}\sum_{a=1}^{N}[\Box_x \imath\Delta_{aa}(x;x')]_{x'\to x} - V_{\rm MF}\right] . \quad (24)$$

Varying the action with respect to $\phi_a(x)$ and $\Delta_{ba}(x';x)$ gives the following (mean field) equations of motion (*cf.* Eqs. (7–8)),

$$\left[\Box_x - m_0^2 - \frac{\lambda}{N}\sum_{b=1}^{N}\left(\phi_b^2 + \imath\Delta_{bb}(x;x)\right)\right]\phi_a(x) - \frac{2\lambda}{N}\sum_{b=1}^{N}\imath\Delta_{ab}(x;x)\phi_b(x) \;=\; 0 \quad (25)$$

$$\left[\Box_x - m_0^2 - \frac{\lambda}{N}\sum_{c=1}^{N}\left(\phi_c^2 + \imath\Delta_{cc}(x;x)\right)\right]\imath\Delta_{ab}(x;x')$$

$$-\frac{2\lambda}{N}\sum_{c=1}^{N}\left[\phi_a(x)\phi_c(x) + \imath\Delta_{ac}(x;x)\right]\imath\Delta_{cb}(x;x') \;=\; \delta_{ab}\frac{\imath\delta^D(x-x')}{\sqrt{-g}} \, , \quad (26)$$

where

$$\imath\Delta_{cb}(x;x') = \left\langle \Omega|T[\delta\hat{\phi}_b(x')\delta\hat{\phi}_a(x)]|\Omega\right\rangle \quad (27)$$

denotes the time-ordered scalar field propagator for scalar field fluctuations, $\delta\hat{\phi}_a(x) = \hat{\phi}_a(x) - \langle\Omega|\hat{\phi}_a(x)|\Omega\rangle$. Just as in the one scalar case, Eqs. (26) can be renormalised by absorbing the infinite part of the coincident scalar propagator (59) into the bare mass m_0^2,

$$m^2 = m_0^2 + \frac{(N+2)\lambda}{N}\frac{H^{D-2}\Gamma\left(\frac{D-1}{2}\right)}{4\pi^{(D-1)/2}}\left[\psi\left(\frac{D}{2}\right) - \psi(D-1) - \psi\left(1-\frac{D}{2}\right) - \gamma_E + \frac{1}{D-1}\right] , \quad (28)$$

resulting in manifestly finite renormalised equations analogous to Eqs. (7–8).

In the limit when the fields are slowly varying ($\Box\phi_a \simeq 0$), the renormalised form of Eq. (25) yields the following criterion for symmetry breaking,

$$\left[m^2 + \frac{\lambda}{N}\sum_{b=1}^{N}\left(\phi_b^2 + \imath\Delta_{bb}(x;x)_{\rm fin}\right)\right]\phi_a(x) + \frac{2\lambda}{N}\sum_{b=1}^{N}\imath\Delta_{ab}(x;x)_{\rm fin}\phi_b(x) = 0 \, . \quad (29)$$

The field mass matrix M^2 is obtained by taking a second field derivative of $V_{\rm MF}$ (or equivalently by taking a single derivative with respect to $\imath\Delta_{ba}$),

$$M_{ab}^2 = \left[m^2 + \frac{\lambda}{N}\sum_{c=1}^{N}\left(\phi_c^2 + \imath\Delta_{cc}(x;x)_{\rm fin}\right)\right]\delta_{ab} + \frac{2\lambda}{N}\left[\phi_a\phi_b + \imath\Delta_{ab}(x;x)_{\rm fin}\right] . \quad (30)$$

Notice that this result can be read off also from Eq. (26), as M^2 is the mass term of the propagator $\imath\Delta_{ab}$. Both Eq. (29) and (30) contain in general off-diagonal terms. One can diagonalize them by

an $N\times N$ dimensional orthonormal matrix $R=(R_{ab})$, $R\cdot R^T=I$, for which

$$\phi^d_a=\sum_b R_{ab}\phi_b=\phi\begin{pmatrix}1\\0\\ \vdots\\0\end{pmatrix}\,,\qquad \imath\Delta^d_{ab}=\sum_{ce}R_{ac}\imath\Delta_{ce}R_{be}=\begin{pmatrix}\imath\Delta^d_{11} & 0 & 0 & \cdots & 0\\ 0 & \imath\Delta^d_{22} & 0 & \cdots & 0\\ \vdots & \vdots & \ldots & \vdots & \vdots\\ 0 & 0 & \ldots & 0 & \imath\Delta^d_{NN}\end{pmatrix}\,. \tag{31}$$

Because of the unbroken $O(N-1)$ symmetry, $\imath\Delta^d_{ii}$ are all equal for $2\le i\le N$. The diagonal form of Eqs. (29–30) is:

$$\Big[m^2+\frac{\lambda}{N}\big(\phi^2+3\imath\Delta^d_{11}(x;x)_{\rm fin}+(N-1)\imath\Delta^d_{22}(x;x)_{\rm fin}\big)\Big]\phi^d_1(x)=0\,. \tag{32}$$

($\phi^d_i=0$ for $2\le i\le N$) and

$$\begin{aligned}M_1^2\equiv(M^d_{11})^2 &= m^2+\frac{\lambda}{N}\left[3\phi^2+3\imath\Delta^d_{11}(x;x)_{\rm fin}+(N-1)\imath\Delta^d_{22}(x;x)_{\rm fin}\right]\\ M_g^2\equiv(M^d_{ii})^2 &= m^2+\frac{\lambda}{N}\left[\phi^2+\imath\Delta^d_{11}(x;x)_{\rm fin}+(N+1)\imath\Delta^d_{22}(x;x)_{\rm fin}\right]\,,\qquad (2\le i\le N)\,.\end{aligned} \tag{33}$$

Eq. (32) implies that the $O(N)$ symmetry is broken when

$$\phi^2=(\phi^d_1)^2=-\frac{Nm^2}{\lambda}-3\imath\Delta^d_{11}(x;x)_{\rm fin}-(N-1)\imath\Delta^d_{22}(x;x)_{\rm fin}>0\,. \tag{34}$$

Otherwise, $\phi^d_1=0$ and the symmetry is unbroken. When this is inserted into (33), we get that the mass terms (in the broken phase) become,

$$\begin{aligned}M_1^2\equiv(M^d_{11})^2 &= \frac{2\lambda}{N}\phi^2=-2m^2-\frac{2\lambda}{N}\left[3\imath\Delta^d_{11}(x;x)_{\rm fin}+(N-1)\imath\Delta^d_{22}(x;x)_{\rm fin}\right]\\ M_g^2\equiv(M^d_{ii})^2 &= \frac{2\lambda}{N}\left[\imath\Delta^d_{ii}(x;x)_{\rm fin}-\imath\Delta^d_{11}(x;x)_{\rm fin}\right]\,,\qquad (2\le i\le N)\,.\end{aligned} \tag{35}$$

In the special case when $N=1$ the first equation agrees with Eq. (12) (provided, of course, one rescales the λ in (35) as $\lambda\to\lambda/6$).

With this, the renormalised and diagonalised form of Eq. (26) becomes,

$$\left[\Box_x-M_1^2\right]\imath\Delta^d_{11}(x;x')=\frac{\imath\delta^D(x-x')}{\sqrt{-g}}\,,\qquad \left[\Box_x-M_g^2\right]\imath\Delta^d_{ii}(x;x')=\frac{\imath\delta^D(x-x')}{\sqrt{-g}}\quad(2\le i\le N)\,. \tag{36}$$

The implied stability of de Sitter space then demands that both $(M^d_{ii})^2=M_g^2>0$ ($N\ge i\ge 2$) and $M_1^2>0$. Next we insert the coincident propagator (59) into (35) to obtain:

$$\begin{aligned}M_1^2 &= -2m^2-\frac{\lambda H^D\Gamma\left(\frac{D+1}{2}\right)}{N\pi^{(D+1)/2}}\left[\frac{N-1}{M_g^2}+\frac{3}{M_1^2}\right]\\ M_g^2 &= \frac{\lambda H^D\Gamma\left(\frac{D+1}{2}\right)}{N\pi^{(D+1)/2}}\left[\frac{1}{M_g^2}-\frac{1}{M_1^2}\right]\,.\end{aligned} \tag{37}$$

Notice that positivity of M_g^2 implies that $M_1^2 > M_g^2 > 0$. The fact that the Goldstone bosons become massive on de Sitter background is reminiscent of the BEH mechanism, in which the Goldstones are 'eaten up' by the longitudinal degrees of freedom of the $W^\pm$ and Z bosons, thus becoming massive.

Equations (37) are the main result of this work. In order to analyse them, it is convenient to work with the following dimensionless quantities,

$$\mu_1^2 = \frac{M_1^2}{(-2m^2)}\,, \qquad \mu_g^2 = \frac{M_g^2}{(-2m^2)}\,, \qquad \lambda_D = \frac{\lambda H^D \Gamma\left(\frac{D+1}{2}\right)}{N\pi^{(D+1)/2}(-2m^2)^2}\,, \tag{38}$$

after which Eqs. (37) become

$$\mu_1^2 = 1 - \lambda_D\left(\frac{N-1}{\mu_g^2} + \frac{3}{\mu_1^2}\right)\,, \qquad \mu_g^2 = \lambda_D\left(\frac{1}{\mu_g^2} - \frac{1}{\mu_1^2}\right)\,. \tag{39}$$

Before we perform a general analysis of these equations, notice that in the case when $N = 1$, the second equation decouples, and one gets

$$\mu_1^4 - \mu_1^2 + 3\lambda_D = 0\,, \tag{40}$$

whose (physical) root [35] is

$$\mu_1^2 = \frac{1}{2} + \sqrt{\frac{1}{4} - 3\lambda_D}\,. \tag{41}$$

The minimum critical mass is then determined by $\lambda_D < (\lambda_D)_{\rm cr} = 1/12$ $(1 \geq \mu_1^2 \geq (\mu_1^2)_{\rm cr} = 1/2)$, which accords with Eq. (15) (when one takes account of the different definition of λ in the Lagrangians (1) and (22)) [36].

In the general case the gap equations (39) admit a small coupling expansion. Similarly as in the thermal case, the expansion parameter is $\sqrt{\lambda_D}$, and hence non-perturbative,

$$\begin{aligned}
\mu_1^2 &= 1 - (N-1)\sqrt{\lambda_D} - \frac{N+5}{2}\lambda_D - \frac{(N-1)(4N-21)}{8}\lambda_D^{3/2} + \mathcal{O}(\lambda_D^2) \\
\mu_g^2 &= \sqrt{\lambda_D} - \frac{1}{2}\lambda_D - \frac{4N-5}{8}\lambda_D^{3/2} + \mathcal{O}(\lambda_D^2)\,.
\end{aligned} \tag{42}$$

In figure 1 we show the rescaled masses μ_1^2 and μ_g^2 as a function of λ_D. Both, the small coupling series solutions (42) (dashed) as well as the full solutions of (39) (solid) are shown. We have plotted the heavy (Higgs) scalar mass $\mu_1^2 = M_1^2/(-2m^2)$ (the upper curves starting at one when $\lambda_D = 0$) and the pseudo-Goldstone mass $\mu_g^2 = M_g^2/(-2m^2)$ (the lower curves starting at zero when $\lambda_D = 0$) for $N = 1$ (the most extended green curves), $N = 2$ (the intermediate red curves), $N = 4$ (the intermediate blue curves) and $N = 10$ (the most squeezed gray curves). While the approximate solutions (42) continue for a while longer, the exact solutions end suddenly at a critical point, at which a minimum

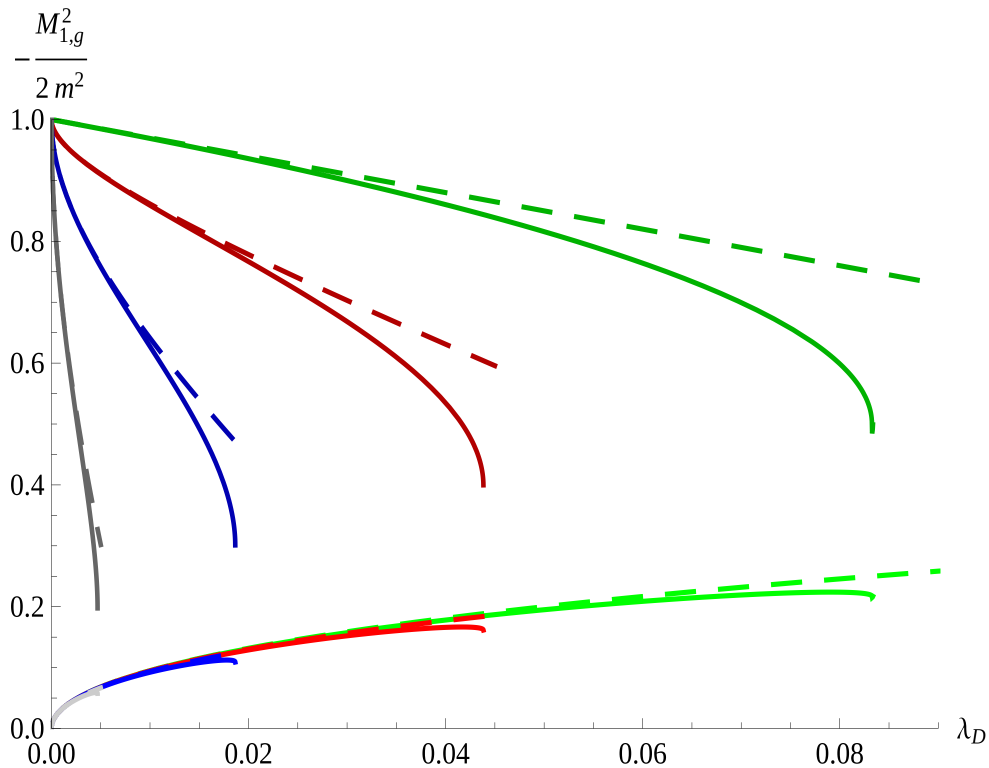

FIG. 1: The rescaled masses $\mu_1^2 = M_1^2/(-2m^2)$ (upper curves) and $\mu_g^2 = M_g^2/(-2m^2)$ (lower curves) as a function of the rescaled (dimensionless) coupling λ_D defined in (38). Both the exact solutions (solid) of Eqs. (39) and the approximate solutions (42) (dashed) are shown. When viewed from the left to the right, we have plotted the cases: $N = 1$ (green), $N = 2$ (red), $N = 4$ (blue) and $N = 10$ (gray).

(critical) mass is reached. Just like for the case when $N = 1$, where the mass parameter decreases monotonically from $\mu_1^2 = 1$ (at $\lambda_D = 0$) to $(\mu_1^2)_{\rm cr} = 1/2$ (at $\lambda_D = (\lambda_D)_{\rm cr} = 1/12$), see Eq. (41), for $N > 1$, μ_1^2 evolves monotonically from $\mu_1^2 = 1$ (at $\lambda_D = 0$) to some $(\mu_1^2)_{\rm cr} > 0$, the end point being a function of N.

As we have seen in the analysis of the one scalar field case, this end point plays an important role as it tells us how the system behaves at the critical point (where the phase transition takes place). In order to study the critical behaviour in some detail, in figure 2 we plot the critical mass parameters $(\mu_1^2)_{\rm cr}$ (upper solid blue curve) and $(\mu_g^2)_{\rm cr}$ (lower dashed red curve) as a function of N (left panel) and as a function of $(\lambda_D)_{\rm cr} = \max[\lambda_D]$ for a fixed N (right panel) (corresponding to the end points on figure 1). One can show that, to good approximation, $(\lambda_D)_{\rm cr} \simeq 3/[2(N+2)^2]$ (more precisely $(\lambda_D)_{\rm cr} \simeq 1.62/[N+2]^{1.92}$), such that $(\lambda_D)_{\rm cr}$ approaches approximately quadratically zero as $N \to \infty$. A manifestation of this is the fractional power behaviour of $\mu_{1,g}^2$ close to the origin for small $(\lambda_D)_{\rm cr}$ (large N) seen on the right panel in figure 2.

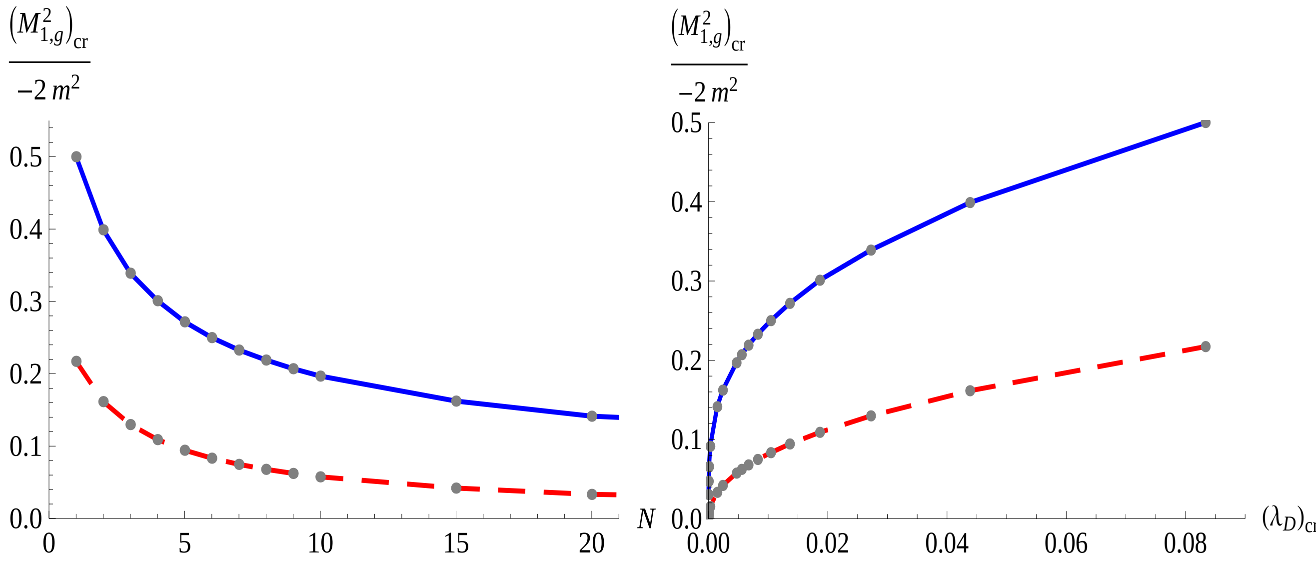

FIG. 2: The rescaled critical (minimum) masses for the Higgs-like excitation $(\mu_1^2)_{\rm cr} = (M_1^2)_{\rm cr}/(-2m^2)$ (the upper solid blue curve) and for the pseudo-Goldstones $(\mu_g^2)_{\rm cr} = (M_g^2)_{\rm cr}/(-2m^2)$ (the lower dashed red curve) as a function of N (left panel) and as a function of the rescaled critical coupling $(\lambda_D)_{\rm cr}$ (defined as the maximum allowed λ_D for a given N) (right panel). Individual points where N is an integer are shown as gray dots.

Just as in the one field case, the phase transition in the $O(N)$ model is of a first order. Indeed, from Eq. (32) we see that

$$\Delta\phi^2 = \frac{N(M_1^2)_{\rm cr}}{2\lambda} = \frac{N(-m^2)(\mu_1^2)_{\rm cr}}{\lambda} > 0\,, \tag{43}$$

with $(\mu_1^2)_{\rm cr}$ plotted in figure 2. In fact, from the left and right panels on figure 2 one can read off that $(\mu_1^2)_{\rm cr} \approx 2/[3\sqrt{N}]$ and $(\mu_1^2)_{\rm cr} \propto (\lambda_D)_{\rm cr}^{1/4} \propto (\lambda/N^2)^{1/4}|m^2|^{-1/2}$, implying that $\Delta\phi^2 \propto |m|N^{1/2}/\lambda^{3/4}$, which is to be compared with the single field result (20), where we found that $(\Delta\phi^2)_{N=1} \propto \lambda^{-1/2}$. Hence, in the large N limit the strength of the phase transition (43) exhibits a qualitatively different dependence on λ and m^2 than in the single field case (20).

For completeness, we shall now briefly analyse the unbroken symmetry case. In this case $\phi_i^d = 0$ $(i = 1, 2, .., N)$ and Eq. (32) implies,

$$m^2 + \frac{\lambda}{N}\left(3\imath\Delta_{11}^d(x;x)_{\rm fin} + (N-1)\imath\Delta_{22}^d(x;x)_{\rm fin}\right) > 0 \tag{44}$$

and analogous steps as above yield,

$$\mu_1^2 = -\frac{1}{2} + \frac{\lambda_D}{2}\left(\frac{N-1}{\mu_g^2} + \frac{3}{\mu_1^2}\right)\,, \qquad \mu_g^2 = \mu_1^2 + \lambda_D\left(\frac{1}{\mu_g^2} - \frac{1}{\mu_1^2}\right)\,. \tag{45}$$

Just as in the broken case, when $N = 1$ we have,

$$\mu_1^2 = -\frac{1}{4} + \sqrt{\frac{1}{16} + \frac{3\lambda_D}{2}}\,, \tag{46}$$

which agrees with Eq. (18). In fact, it is quite easy to obtain the general solution of equations (45). Indeed, observe that the second equation can be written as,

$$(\mu_1^2 - \mu_g^2)(\lambda_D + \mu_1^2\mu_g^2) = 0\,. \tag{47}$$

The positivity of λ_D, μ_1^2 and μ_g^2 immediately implies that the only consistent solution is

$$\mu_g^2 = \mu_1^2\,. \tag{48}$$

It is not surprising that in this case all particles must have the same mass since the symmetry is unbroken. With this, the first equation in (45) is easily solved,

$$\mu_1^2 = -\frac{1}{4} + \sqrt{\frac{1}{16} + \frac{(N+2)\lambda_D}{2}}\,, \tag{49}$$

which can be also written as,

$$M_1^2 = M_g^2 = \frac{m^2}{2} + \sqrt{\frac{m^4}{4} + \frac{(N+2)m_{\rm cr}^2}{6}}\,, \tag{50}$$

where $m_{\rm cr}$ is given in (15). This generalizes the real field result (18) to the $O(N)$ symmetric case. From Eq. (50) one can easily see that $M_1^2 = M_g^2 > 0$, as it should be. Expanding the solution (49) in powers of λ_D, we see that – unlike in the broken case (42) – the result (49) is perturbative (and analytic) in λ_D, $\mu_1^2 = \mu_g^2 = (N+2)\lambda_D - 2(N+2)^2\lambda_D^2 + \mathcal{O}(\lambda_D^3)$.

The same criticism of validity of the adiabatic approximation used in this section applies to the $O(N)$ model as it did to the real scalar field model (see the end of section II).

IV. DISCUSSION

We have analysed the $O(N)$ symmetric scalar field model (22) in the mean field approximation (23) on de Sitter space. We have shown that symmetry breaking can occur, and that the would-be Goldstone bosons acquire a mass (see figure 1) due to the enhanced infrared correlations in de Sitter space. Next we have studied the strength of the symmetry breaking transition, and we have shown that the (phase) transition is always of a first order. Curiously, the jump in the order parameter (43) exhibits a non-analytic dependence on the parameters of the model,

$\Delta\phi^2 \propto |m| N^{1/4}\lambda^{-3/4}$, where $|m|$ and λ denote the mass parameter and the quartic coupling of the model.

While the mean field results are of their own interest, it would be desirable to investigate whether (and how) the mean field results presented here change when one includes higher loop corrections. A first step in this direction is taken in Ref. [24] where the local contribution to the self-mass from the two loop (sun-set) diagram was estimated, and where it was found that, in the massless limit, the mean field mass-squared gets reduced by a factor $1/\sqrt{2}$.

Second, it is instructive to compare our results with the (old) stochastic theory results of Starobinsky and Yokoyama [28], which is known to resum the leading $\log(a)$ corrections to infrared correlators on de Sitter space, see *e.g.* [9]. From Eq. (23) of Ref. [28] we read (upon a rescaling, $\lambda \to \lambda/6$),

$$m^2_{\rm stoch} = \frac{\lambda}{2}\langle\phi^2\rangle = \frac{3}{2\pi}\frac{\Gamma(3/4)}{\Gamma(1/4)}\sqrt{\lambda}H^2 \approx 0.1614\sqrt{\lambda}H^2\,, \tag{51}$$

which is to be compared with Eqs. (18) and (15), which in the limit when $m^2 \to 0$ and in $D = 4$ yield $m^2_{\rm MF} \to \sqrt{3\lambda}H^2/(4\pi)$. This then implies,

$$\frac{m^2_{\rm stoch}}{m^2_{\rm MF}} = 2\sqrt{3}\frac{\Gamma(3/4)}{\Gamma(1/4)} \approx 1.17\,. \tag{52}$$

Even though the difference in the results is modest, the question – which result is correct? – is, nevertheless, important. In the derivation of the stochastic result (51), one assumes that the tree level potential remains unchanged, *i.e.* that for the late time behaviour the tree level potential should be used when stochastic theory is applied to inflation. At the moment there is no fundamental understanding concerning whether the tree level potential or some effective potential should be used in stochastic formalism. We close this discussion by noting that one can recover exactly the mean field result $(m^2_{\rm MF})_{m\to 0}$ from stochastic formalism, provided one replaces the tree level potential $V = (\lambda/4!)\phi^4$ by its Gaussian counterpart, $V \to V_{\rm Gauss} = (\lambda/4)\langle(\phi)^2\rangle\phi^2$. While this is suggestive, it does not ultimately tell us what is the correct answer.

Acknowledgments

I thank Bjorn Garbrecht, Louis Leblond, Albert Roura, Julien Serreau and Richard Woodard for useful discussions. I am particularly grateful to Louis and Albert for very useful discussions at the 'Quantum Gravity: From UV to IR' workshop at CERN, during which the basic idea of the paper was born.

Appendix: The de Sitter space propagator for massive and massless scalar fields

Here we review some of the basic properties of the scalar propagator on de Sitter background in D space time dimensions. The time ordered (Feynman) propagator $\imath\Delta \equiv \imath\Delta^{++}$ obeys the equation,

$$\sqrt{-g}\left[\Box_D - m^2\right]\imath\Delta^{++}(x;x') = \imath\delta^D(x-x')\,, \tag{53}$$

where m is a mass and $\Box_x = (-g)^{-1/2}\partial_\mu g^{\mu\nu}\sqrt{-g}\partial_\nu$ is the scalar d'Alembertian in D spacetime dimensions. The de Sitter invariance allows one to write the d'Alembertian in a de Sitter invariant form,

$$\Big[\bar{y}(4-\bar{y})\frac{d^2}{d\bar{y}^2} + D(2-\bar{y})\frac{d}{d\bar{y}} - \frac{m^2}{H^2}\Big]\imath\Delta(x;x') = \frac{\imath}{\sqrt{-g}H^2}\delta^D(x-x')\,, \tag{54}$$

where $\bar{y} = a(\eta)a(\eta')H^2[-(\eta-\eta')^2 + \|\vec{x}-\vec{x}'\|^2]$ is related to the geodesic distance on de Sitter space $\ell(x;x')$ as, $\bar{y} = 4\sin^2(H\ell/2)$. Here a denotes the scale factor,η is conformal time and $\vec{x}$ comoving coordinate. The unique solutions for the relevant propagators of the Schwinger-Keldysh (or *in-in*) formalism can be written in terms of the Gauss' hypergeometric function ${}_2F_1$ as follows,

$$\imath\Delta^{\alpha\beta}(x;x') = \frac{H^{D-2}}{(4\pi)^{D/2}}\frac{\Gamma(\frac{D-1}{2}+\nu_D)\Gamma(\frac{D-1}{2}-\nu_D)}{\Gamma(\frac{D}{2})}\times{}_2F_1\Big(\frac{D-1}{2}+\nu_D, \frac{D-1}{2}-\nu_D; \frac{D}{2}; 1-\frac{y^{\alpha\beta}}{4}\Big)\,, \tag{55}$$

where

$$\nu_D^2 = \Big(\frac{D-1}{2}\Big)^2 - \frac{m^2}{H^2}\,. \tag{56}$$

Here $m^2 > 0$ represents the (renormalised) field mass parameter, which includes the renormalised mass, the mean field correction (the finite part of $(\lambda/2)\imath\Delta(x;x)$) and possibly also the term that originates from a nonminimal coupling, $\Delta m^2 = \xi D(D-1)H^2$ in the lagrangian, $\Delta\mathcal{L} = -\xi R\phi^2$, where $R = D(D-1)H^2$ is the Ricci scalar in de Sitter space. The functions $y^{\alpha\beta}$ $(\alpha,\beta = \pm)$ in Eq. (55) denote,

$$\begin{aligned}
y^{++} &= a(\eta)a(\eta')H^2[-(|\eta-\eta'|-\imath\epsilon)^2 + \|\vec{x}-\vec{x}'\|^2] \\
y^{+-} &= a(\eta)a(\eta')H^2[-(\eta-\eta'+\imath\epsilon)^2 + \|\vec{x}-\vec{x}'\|^2] \\
y^{-+} &= a(\eta)a(\eta')H^2[-(\eta-\eta'-\imath\epsilon)^2 + \|\vec{x}-\vec{x}'\|^2] \\
y^{--} &= a(\eta)a(\eta')H^2[-(|\eta-\eta'|+\imath\epsilon)^2 + \|\vec{x}-\vec{x}'\|^2]\,,
\end{aligned} \tag{57}$$

with $\epsilon > 0$ infinitesimal. All propagators in (55) have the same coincident limit,

$$\imath\Delta(x;x) = \imath\Delta^{\alpha\beta}(x;x) = \frac{H^{D-2}}{(4\pi)^{D/2}}\frac{\Gamma\left(\frac{D-1}{2}+\nu_D\right)\Gamma\left(\frac{D-1}{2}-\nu_D\right)}{\Gamma\left(\frac{1}{2}+\nu_D\right)\Gamma\left(\frac{1}{2}-\nu_D\right)}\Gamma\left(1-\frac{D}{2}\right)\,. \tag{58}$$

Due to the last Γ function, this propagator exhibits a simple pole in even dimensions, $D = 2, 4, 6, ..$, which reflects an ultraviolet (UV) logarithmic divergence. Of course, the leading UV divergence of the coincident propagator in de Sitter space is the same as that in Minkowski space, and it is of a degree $D-2$, the subleading is of a degree $D-4$, *etc.*, the degree zero representing a logarithmic divergence. As it is well known, dimensional regularisation is blind to power law divergences (they are automatically subtracted by analytic extension), and exhibits only logarithmic divergences. The effect of the propagator (58) can be considered in the weak curvature (Minkowski) limit, when $m^2 \gg H^2$ (in which case one recovers the Minkowski space result plus small corrections) and in a strong curvature regime, in which $m^2 \ll H^2$. Ignoring the UV divergence in (58) one can naively expand it in powers of m^2/H^2, and one obtains [37],

$$\begin{aligned} \imath\Delta(x;x) &= \frac{H^{D-2}\Gamma\left(\frac{D-1}{2}\right)}{4\pi^{(D-1)/2}}\left[\psi\left(\frac{D}{2}\right)-\psi(D-1)-\psi\left(1-\frac{D}{2}\right)-\gamma_E+\frac{1}{D-1}\right] \\ &+ \frac{\Gamma\left(\frac{D+1}{2}\right)}{2\pi^{(D+1)/2}}\frac{H^D}{m^2}+\mathcal{O}\Big(\frac{m^2}{H^2}\Big)\,, \end{aligned} \tag{59}$$

such that in $D = 2, 3, 4$ the $\mathcal{O}(m^{-2})$ terms are $H^2/(4\pi m^2)$, $H^3/(2\pi^2 m^2)$, and $3H^4/(8\pi^2 m^2)$, respectively. In our analysis in the main text we assume that both finite and infinite m-independent terms in (59) are absorbed in the physical definition of the mass term.

The de Sitter invariant limit will be attained after some time during inflation. If the mass is very small ($m^2 \ll H^2$), the propagator will at early times grow logarithmically with the scale factor (linearly with cosmological time). This can be seen by recalling that in the infrared [11] the coincident propagator satisfies,

$$\begin{aligned} \imath\Delta(x;x) &= \frac{H^{D-2}}{2^D\pi^{(D-3)/2}\Gamma\left(\frac{D-1}{2}\right)}\int_{k_0/(Ha)}^{\infty} dz z^{D-2}|H^{(1)}_{\nu_D}(z)|^2 \\ &= \frac{H^{D-2}}{(4\pi)^{D/2}}\frac{\Gamma\left(\frac{D-1}{2}+\nu_D\right)\Gamma\left(\frac{D-1}{2}-\nu_D\right)}{\Gamma\left(\frac{1}{2}+\nu_D\right)\Gamma\left(\frac{1}{2}-\nu_D\right)}\Gamma\left(1-\frac{D}{2}\right)-\frac{H^{D-2}\Gamma\left(\nu\right)^2}{8\pi^{(D-3)/2}\Gamma\left(\frac{D-1}{2}\right)}\times\frac{\left(\frac{k_0}{2Ha}\right)^{D-1-2\nu_D}}{D-1-2\nu_D} \\ &\quad +\mathcal{O}\left(k_0^{D+1-2\nu_D}, k_0^{D-1}, k_0^{D-1+2\nu_D}\right)\,, \end{aligned} \tag{60}$$

where we took $a(t_0) = a_0 = 1$ and k_0 is an infrared (comoving) momentum cut-off. Notice that at early times (and in the limit when $m \to 0$ and $\nu_D \to (D-1)/2$) the coincident propagator grows logarithmically with time as (see also Refs. [29, 30])

$$\begin{aligned} \imath\Delta(x;x) &= \frac{H^{D-2}\Gamma\left(\frac{D-1}{2}\right)}{4\pi^{(D-1)/2}}\left[\psi\left(\frac{D}{2}\right)-\psi(D-1)-\psi\left(1-\frac{D}{2}\right)-\gamma_E+\frac{1}{D-1}\right] \\ &+ \frac{H^{D-2}\Gamma\left(\frac{D-1}{2}\right)}{2\pi^{(D+1)/2}}\left[\ln(a)-\ln\left(\frac{k_0}{2H}\right)\right]+\mathcal{O}(m^2/H^2)\,. \end{aligned} \tag{61}$$

This is to be compared with the Onemli-Woodard coincident propagator for a massless scalar field [6] [38]:

$$[\imath\Delta(x;x)]_{\rm OW} = \frac{H^{D-2}}{4\pi^{(D-1)/2}} \frac{\Gamma\left(\frac{D}{2}\right)\Gamma\left(1-\frac{D}{2}\right)}{\Gamma\left(\frac{3-D}{2}\right)} + \frac{H^{D-2}\Gamma\left(\frac{D-1}{2}\right)}{2\pi^{(D+1)/2}}\ln(a)\,. \tag{62}$$

The logarithmic growth saturates when the propagator reaches the de Sitter invariant value (59), which characterizes the time scale at which the propagator (and thereby the state) becomes de Sitter invariant.

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[31] As early as 1986 Ford and Vilenkin [19] correctly reasoned: "[..] it would be completely consistent with all observations for our present Universe to be a de Sitter space with $H^{-1} \sim 10^{10}$ yr. It would be very surprising if this were to have any effect upon symmetry breaking on terrestrial or subatomic scales." While this observation is correct, it does not follow from their analysis.

[32] To renormalise the mean field effective potential (2), one also needs to renormalise V_0. Since V_0 does not affect our analysis, we shall not renormalise V_0 here.

[33] A simple algebra shows that both mean field masses-squared $m^2_{\rm MF\pm}$ in (14) are positive, and moreover both are consistent with broken symmetry $\phi^2 > 0$, see (11), which is to be contrasted to the conclusion in [23].

[34] Equation (18) has already been derived in Refs. [23, 24], where the sign error in [18] in front of λ was corrected.

[35] Recall that, in the limit when $\lambda_D \to 0$, the physical branch yields $\mu_1^2 = M_1^2/(-2m^2) = 1$.

[36] Formally, one can also solve Eq. (39) for μ_g^2 in the $N = 1$ case, and one finds for the critical value, $(\mu_g^2)_{\rm cr} = (\sqrt{13} - 1)/12 \simeq 0.217$, which agrees with the results plotted in figures 1 and 2.

[37] The $\mathcal{O}(m^2)$ term in Eq. (59) has a divergent coefficient. To make the renormalised gap equation consistent, the divergent part of the $\mathcal{O}(m^2)$ term would have to be absorbed in the renormalised mass term m^2.

[38] Not surprisingly, the coincident propagator of a light scalar field (60) and the Onemli-Woodard coincident massless scalar propagator (61) possess identical late time logarithmically growing terms $\propto \ln(a)$. The time-independent parts do not agree, however. But this was to be expected, since these constant pieces do not have an independent physical meaning, as they can be absorbed in the mass counterterm of the self-interacting scalar theory.